The Housing Bubble and a New Approach to Accounting for Housing in a CPI

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Abstract

Over the course of the recent house price bubble in the United States, the price of homes rose rapidly from 1999 Q4 to 2005 Q4 (11.3 % annually as measured by the Case-Shiller index, and 8.4 percent annually as measured by the Federal Housing Financing Agency) but slowly as measured by owner equivalent rents (3.4 percent), so measured core inflation remained relatively docile during this period since only rents are used to measure inflation for housing services in the United States. Over the last several decades, the U.S. Bureau of Labor Statistics (BLS) has experimented with both rental equivalence and user cost approaches for accounting for owner occupied housing (OOH) services in the CPI. We explain the basics of these approaches, and outline the BLS experiences with using them. This assessment leads us to conclude that the time has come to try a new approach: the opportunity cost approach. We argue this approach has advantages over both the conventional rental equivalence and user cost approaches, though it embeds components of the measures for both those approaches and builds solidly on the research of Verbrugge and others at the BLS. Also, we take up empirical issues that must be faced regardless of which of the approaches discussed is adopted. We explain how the repeat-sales and various hedonic regression methods can be placed in a common framework, thereby facilitating understanding of the properties of and the tradeoffs between the methods. We also consider measurement complications that arise because the land and structure components of properties depreciate at different rates.

1. Introduction

How is the cost of housing services changing over time for those living in their own homes? Good measures are needed by economic policy makers managing everything from the money supply to benevolent income transfer programs, but are hard to come by since homeowners do not actually pay themselves for the services of their owned homes.

Over the course of the recent house price bubble in the United States, the price of homes rose rapidly from 1999 Q4 to 2005 Q4 (11.3 % annually as measured by the Case-Shiller index, and 8.4 percent annually as measured by the Federal Housing Financing Agency) but slowly as measured by owner equivalent rents (3.4 percent). One consequence was that measured core inflation remained relatively docile during this period since only rents are used to measure inflation for housing services in the United States. Yet, as Gallin (2009) and Crone et al. (2009)
have argued, rents and home prices are cointegrated over the long run. Moreover, Gallin has presented evidence that home prices tend to overshoot while rents tend to lag.

If the housing bubble took the form of unsustainable increases in home prices, the expectation that these increases could be sustained in the short run perhaps lead to a short run divergence between rents and home prices. If there is information about inflation trends in both rents and home prices, it might be useful to combine both types of measures. The opportunity cost approach which we develop combines information on home price change, as part of a financial user cost component, and information on rents as a rental equivalent component.

Over the last several decades, the U.S. Bureau of Labor Statistics (BLS) has experimented with both rental equivalence and user cost approaches for accounting for owner occupied housing (OOH) services in a Consumer Price Index (CPI). We explain the basics of these approaches in sections 2 and 3, respectively, and outline the BLS experiences with using them in section 4. This assessment leads us to conclude that the time has come to try a new approach to accounting for OOH services costs in measures of inflation: a new approach that nevertheless builds on BLS expertise and research findings, especially including the work of Randall Verbrugge and his collaborators.2

The opportunity cost approach, introduced in section 5, was first suggested at a 2006 OECD Workshop by Erwin Diewert.3 We argue this approach has advantages over both the conventional rental equivalence and user cost approaches, though it embeds components of the measures for both those approaches. Also, in sections 6 and 7, we take up empirical issues that must be faced regardless of which of the approaches discussed is adopted. We explain how the repeat-sales and various hedonic regression methods can be placed in a common framework, thereby facilitating understanding of the properties of and the tradeoffs between the methods. We also consider measurement complications that arise because the land and structure components of properties depreciate at different rates. Section 8 concludes.

2. The Rental Equivalence Approach

The rental equivalence approach values the services yielded by an owned dwelling at the corresponding market rental value for the same sort of dwelling for the same period of time. This is the approach used by the BLS at present for the CPI.4 The price data needed for the CPI rental equivalence component for OOH services are observations on rents paid by renters: the same price data also used by the BLS to compile the rental component of the CPI.

The location of each rental unit for which rent and other data are collected is unique. Empirical studies have shown location to be a key determinant not only of both rents and residential real estate price levels, but also of the rates of change over time in the levels. Hence, after choosing a sample of dwelling units to use for the collection of rent data, the BLS repeatedly samples those units. It is assumed that the changes in owners’ equivalent rents within small geographic areas (areas of 3-4 city blocks, sometimes called segments) will move similarly to changes in actual rents. (The nature of this rent data, and some of the main data sets for

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2 See Verbrugge (2008) and Garner and Verbrugge (2009), and also Poole, Ptacek and Verbrugge (2005).
housing price data too, are why, in section 6, we explore the relationship between the repeat sale and hedonic estimation methods.) Each rental unit that is priced does double duty: it represents the rents for renters within the segment, and it also separately represents the rent equivalents implicitly paid by owners within the segment.

3. **The User Cost Approach**

The only nations that use the user cost approach to account for the cost of OOH services in their official measures of inflation omit the property appreciation term. However, reports on the treatment of OOH by official statistics agencies, including the BLS, make frequent reference to the shared theoretical underpinnings for the user cost and the rental equivalency approaches, and it is the user cost, including the property appreciation term, that is relevant in this regard. The property appreciation term of the user cost formula also plays an important role in the research of Verbrugge and his collaborators. Hence, in this section, we describe the user cost approach and show why and how the property appreciation term enters into the user cost formula.

The user cost approach is routinely used in a variety of other measurement and accounting contexts too, such as in the capital asset pricing literature, in production function studies, in the measurement of total factor productivity growth, and in the analysis of tax depreciation rules. The underlying theoretical framework is provided by the fundamental equation of capital theory. According to this equation, in equilibrium, the price of a durable asset equals the present discounted value of the future net income that is expected to be derived from owning it. Thus, if the future income flow that an asset such as a machine can generate is known or can be readily forecast, then this information can be used to infer what the asset would be worth to a buyer. On the other hand, in the literature on inflation measurement for OOH services, what is directly observed are the purchase prices for houses and there are no observable transactions for the rent that owner occupiers implicitly charge themselves for use of their homes. Instead, the fundamental equation of capital theory is used to try to back out the period by period costs to the owner occupier of the OOH services they are using.

Diewert (1974, p. 504) sets out the user cost principles for consumer durables:6

“To form the rental price (or user cost) for the services of one unit of the nth good during period t, we imagine that the consumer purchases the good during period t and then sells it during the following period (possibly to himself). Then the discounted expected rental price for the nth consumer good during period t is given by the discounted cost of the purchase of the nth good during period t minus the discounted resale value of the depreciated good during period t + 1.”

And, Bajari, Benkard, and Krainer (2003, p. 3) outline the connection in the housing economics literature of user costs to the rental equivalence approach:

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5 See Diewert and Nakamura (2009) for summary information and references regarding the use of the user cost approach by Statistics Iceland and Statistics Canada.

“Dougherty and Van Order (1982) were among the first to recognize that the user cost ... should be equal to the rental price of a single unit of housing services charged by a profit-maximizing landlord. Thus, the inherently difficult task of measuring an unobservable marginal rate of substitution is replaced by the much easier task of measuring rents.”

3.1 The Fundamental Equation of Capital Theory

A clear understanding of the basics of the financial theory justification conventionally cited as the basis for both the rental equivalence and the user cost approaches is helpful to have before we discuss the opportunity cost approach. We review this financial theory basis here.

Attention to timing matters for understanding user costs. Realized prices are determined at points in time. Rates of interest are regarded as fixed at points in time. In contrast, rates of inflation are defined for time intervals. If there is inflation, money is less valuable when received at the end versus the beginning of a period. An end of period t value can be converted to its equivalent at the beginning of that same (not the next) period by discounting by the term $1 + r^t$, where $r^t$ is the period t nominal interest rate.

As Arnold Katz (2009) of the U.S. Bureau of Economic Analysis (BEA) explains, the “user cost of capital” is based on the fundamental equation of capital theory. In box 1, the derivation of the user cost by Katz (2009, appendix A) is shown, recast using the notation for our paper. We denote the value of a home that is v periods old at the start of period t by $V^t_V$. Given only the information available at the start of t, the expected price a home could be sold for at the end of period t, which is the start of period $t+1$, is denoted by $V^{t+1}_{V+1}$. $O^t_V$ denotes the anticipated operating costs including maintenance expenses.

Katz explains that the user cost measure is typically derived by assuming that flow transactions within a period occur at the end of the period. Following this convention leads to the end of period user cost, shown in box 1 as equation (3-4):$u^t_V = r^t V^t_V + O^t_V - (V^{t+1}_{V+1} - V^t_V)$.

The symbol m in box 1 (above expression (3-1)) denotes the remaining service life of the home. The expected market value of a home at the end of period t ($V^{t+1}_{V+1}$) is conditional on the home having a remaining service life of m periods.

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7 The BEA makes use of the OOH component of the CPI to supplement the other information they use for accounting for OOH inflation in compiling the National Income and Product Accounts for the United States.
8 Diewert (2005a, 2005bb) also carefully distinguishes between beginning and end of period user costs and recommends the use of end of period user costs since they are more consistent with financial accounting conventions.
9 Unlike the home value variable where we need to refer to both the beginning and the end of period values, we only need to refer to the end of period values for the other anticipated variables and denote them simply using t as the superscript, as Katz does. We also forego using a special designation for expected values.
Box 1. Derivation of the User Cost Measure from Katz (2008, Appendix A)

The user cost of capital measure provides an estimate of the market rental price based on costs of owners. It is directly derived from the principle that, in equilibrium, the purchase price of a durable good will equal the discounted present value of its expected net benefits; i.e., it will equal the discounted present value of its expected future services less the discounted present value of its expected future operating costs. To see this, let $V_t^v$ denote the purchase price of a $v$ periods old durable at the beginning of period $t$; $V_{t+1}^{v+1}$ denote its expected purchase price at the beginning of period $t+1$ when the durable is one period older; $u_t^v$ denote the expected end of period value of the period $t$ services of this durable; $O_t^v$ denote the expected period $t$ operating expenses to be paid at the end of period $t$, for this $v$ periods old durable in period $t$; and $r_t$ denote the expected nominal discount rate (i.e., the rate of return on the best alternative investment) in period $t$. Expected variables are measured as of the beginning of period $t$.

Assume that the entire value of the durable’s services in any period will be received at the end of the period, and that the durable is expected to have a service life of $m$ periods.

From the definition of discounted present value,

$$\sum_{i=1}^{m} (1+r_t)^{-(i+1)} u_t^{v+i} - \sum_{i=1}^{m} (1+r_t)^{-(i+1)} O_t^{v+i} = 0$$

When the durable is one period older, the services it renders in period $t$ will have been received and the operating expenses of period $t$ already incurred. Thus, the expected price of the durable at the beginning of period $t+1$ is:

$$V_{t+1}^{v+1} = \frac{u_{t+1}^{v+1}}{1+r_t} + \frac{u_{t+2}^{v+1}}{(1+r_t)^2} + \ldots + \frac{u_{t+m-v-1}^{v+1}}{(1+r_t)^{m-v}} - \frac{O_{t+1}^{v+1}}{1+r_t} - \frac{O_{t+2}^{v+1}}{(1+r_t)^2} - \ldots - \frac{O_{t+m-v-1}^{v+1}}{(1+r_t)^{m-v}}.$$

Dividing both sides of (3-2) by $(1+r_t)$, subtracting the result from equation (3-1), and rearranging terms yields

$$V_t^v - \frac{V_{t+1}^{v+1}}{1+r_t} = \frac{u_t^v}{1+r_t} - \frac{O_t^v}{1+r_t}.$$

Multiplying through equation (3-3) by $(1+r_t)$ and rearranging terms, then yields the end of period $t$ user cost:

$$u_t^v = r_t V_t^v + O_t^v - (V_{t+1}^{v+1} - V_t^v).$$

### 3.2 The Verbrugge Variant (VV) of the User Cost Approach

Randall Verbrugge, in research of his own and with various colleagues, has sought to determine whether rents and user costs move together, as the financial economics theory outlined in box 1 seems to imply should be the case. We refer to the specification of the user cost implemented in Poole, Ptacek and Verbrugge (2005) (PPV hereafter), and that is explained and investigated more fully in Verbrugge (2008) and Garner and Verbrugge (2008 and 2009), as the Verbrugge variant user cost: the VV user cost for short. The VV user cost can be stated as:

$$u_t^v = [r_t + \gamma_t^v + E_t^v] V_t^v,$$

where $V_t^v$ is the beginning of period home value; $r_t$ is a nominal interest rate; $\gamma_t^v$ collects ongoing “housekeeping” operating expenses; and $E_t^v$ is an estimate of the expected home price appreciation. Note that (3-5) is essentially the same as formula (3-4) in box 1.
What distinguishes the VV user cost from the generic one is that Verbrugge uses alternative forecasting equations for the $E \pi$ term in (3-5). In his preferred equation, documented most fully in Verbrugge (2008) and Garner and Verbrugge (2009), the term $E \pi$ is specified to be the expected value of the 4-quarter home price appreciation. With this setup, changes in home prices have an immediate within-year impact on the user cost. Verbrugge shows empirically that, since 1998, the VV user cost tracks neither rents nor house prices when evaluated using his preferred forecasting equation.

4. The Rental Equivalence versus the User Cost Approach

The BLS has experimented over the years with both the user cost and rental equivalence approaches. Until the early 1950s, homeowners’ costs to rent were imputed by the BLS: a rental equivalence approach. Dissatisfaction with this approach developed due to widespread rent controls, and this led the BLS to switch to what came to be called the “Asset Price” approach. With this simplified user cost approach, which Greenlees (2003) terms an “ad-hoc user cost” approach, OOH services costs within the CPI were built up from five elements: (1) home purchase prices, (2) mortgage interest costs, (3) property taxes, (4) homeowner insurance charges, and (5) maintenance and repair costs. Over time, problems arose with this BLS approach too.

By the early 1980s, the quality of the data available to the BLS on house prices and mortgage interest rates was deteriorating. The source of the house price and mortgage data utilized by the BLS then was the Federal Housing Administration (FHA) administrative database for FHA-insured houses: a small, atypical and shrinking segment of the housing market. Also, the influential Stigler Report (Stigler 1961, p. 53) had come out strongly two decades earlier in favor of rental equivalency. Thus, in 1983 the BLS switched back to a rental equivalence procedure for dealing with OOH services in the CPI. Rents, of course, were being collected all along anyway for the rental component of the CPI.

The rental equivalence approach is still being used. However, PPV note that, by 2005, the rapid rise in housing prices in the post 1999 years coupled with slow increases in the OOH component of the CPI had led to concern among many economic analysts about the use of rental equivalence for accounting for OOH services costs. Younger people, many of whom faced deteriorating employment and earnings outcomes along with rising home prices, were among those who argued that rising housing prices were pushing up their cost of living sharply. Yet, the

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10 The Poole, Ptacek and Verbrugge (2005) paper was prepared for presentation to the U.S. Federal Economic Statistics Advisory Committee (FESAC) on December 9, 2005. The FESAC is a Federal Advisory Committee sponsored jointly by the Bureau of Labor Statistics of the U.S. Department of Labor and by the Bureau of Economic Analysis and the Bureau of the Census of the Department of Commerce.

11 See Poole, Ptacek and Verbrugge (2005). Also, Katz (1982, 1983) at the BEA explored the sensitivity of user cost estimates to alternative assumptions about expected rates of inflation and patterns of depreciation.

12 The Stigler Report (Stigler 1961, p. 53) states that: “The welfare of consumers depends on the flow of services from houses and not upon the stocks acquired in any given period.” The report concluded that (p. 48) “If a satisfactory rent index for units comparable to those that are owner-occupied can be developed, this committee recommends its substitution in the CPI for the present series for the prices of new houses and related expenses.”

rental equivalent component of the CPI rose little for the nation and indeed fell for some urban areas.

Bauer, Haltom, and Peterman (2004) with the Federal Reserve Bank of Atlanta argue that, in the United States, the general softening of rents in 2002-2003 is causally related to increases over the same period in the demand for owned homes. They argue that many of those who had planned to purchase homes over the coming years instead rushed to buy out of fear they would be shut out of the owned housing market by the price increases. Many home buyers entered into mortgage contracts with exceptionally little equity down based on expectations, on their part and held as well by lenders, of rising home values.

Yet, concerning possible alternatives to rental equivalence that the BLS might consider, PPV write that the user cost approach is “the only serious alternative to rental equivalency.” We agree, at least, that neither of the other main approaches currently in use by major official statistics agencies -- namely, the acquisitions and the payments approaches -- would be a suitable alternative.14

We agree too with some of the problems that PPV raise concerning a user cost approach. They note, for example, that the capital theory used to derive the user cost approach only holds under equilibrium conditions. Yet, a housing bubble is a sign that the housing market is not in equilibrium. Furthermore, as conventionally specified, in a period of rapid house price inflation, the appreciation component of the user cost expression15 can grow large enough so that the user cost turns negative. Yet it makes no sense to have a negative figure for the value owner occupiers place on living in their homes!

We conclude that the time has come to find a new approach to accounting for OOH services in a CPI. This is the task we turn our attention to in the following section. The new opportunity cost approach that we recommend builds directly on suggestions made by PPV in their seminal 2005 paper. On an operational level, the proposed approach also builds on the expertise and data collection instruments amassed over the previous decades at the BLS.

5. The Opportunity Cost Approach

As noted in the introduction, in a 2006 address, Diewert recommended an opportunity cost approach for dealing with OOH in a CPI:16

“[P]erhaps the correct opportunity cost of housing for an owner occupier is not his or her internal user cost but the maximum of the internal user cost, which is the financial opportunity cost of housing, and what the property could rent for on the rental market. After all, the concept of opportunity cost is supposed to represent the maximum sacrifice that one makes in order to consume or use some object.”

Diewert and Nakamura (2009, p. 20) followed up with a two-part suggestion for how an opportunity cost approach might be used to compile a price index for OOH services:

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14 See Diewert (2003/2010) for more on this issue.
15 The appreciation component is the final term in parentheses on the right hand side of (3-4) in box 1.
16 See p. 113 in Diewert (2006/2009), which is the published version of Diewert’s presentation at the OECD-IMF Workshop on Real Estate Price Indexes held in Paris, November 6-7, 2006.
For each household living in owner occupied housing (OOH), the opportunity cost is the maximum of what the dwelling could have been rented out for, which is the rental equivalent, and the financial user cost of the funds tied up by owning the property.

The OOHOC index for a nation can be defined as an expenditure share weighted sum of a rental equivalency index and a financial user cost index, with the expenditure share weights depending on the proportion of owner occupiers for whom the financial user cost is estimated to exceed the rental equivalent cost.

Our purpose in this section is to further develop and explore the properties of two components of the proposed OOHOC index. The term “opportunity cost” refers to the net value of the best of the alternatives given up in taking the option chosen. Before proceeding further with the derivation of the household level components of an OOHOC index, in the following section, we briefly discuss the concept of an opportunity cost.

5.1 Opportunity Cost Basics

Consider the example provided in the Wikipedia article on opportunity costs. The example given is for a city that has decided to build a hospital on vacant land it owns. In opting to build a hospital, the city passed up two competing development proposals: (A) a sports arena, and (B) another revenue generating commercial proposal. The Wikipedia write-up states that:

“If the city decides to build a hospital on the vacant land it owns, the opportunity cost is the value of the benefits forgone of the next best thing that might have been done with the land.”

Thus, the opportunity cost is the present value of the greater of the two development opportunities that were passed up in choosing to use the vacant land for the hospital.

In the Wikipedia article, it is explained that the reason that the opportunity cost is the greater -- that is, the maximum -- rather than the sum of the expected outcomes for the two proposals passed up is because those other opportunities and the one chosen were mutually exclusive. Note too that the opportunity cost is being evaluated after the choice to use the land for the hospital has been made. Equivalently, the opportunity cost could have been assessed prior to the decision to go forward with the hospital, but from the perspective that this decision should be adopted. To compute the opportunity cost of some course of action, the analysis must be carried out from the perspective that the designated action was or will be chosen and that the other alternatives were rejected.

In the Wikipedia example, the construction cost of the hospital and an estimate of the market value of the vacant land could be calculated, just as the full purchase price of a home can be observed or estimated. However, another feature of the Wikipedia example that is relevant is that the two options that the city passed up in choosing to build a hospital would both have generated net revenue flows that could be forecasted, and that furthermore should be considered in determining the full economic cost of the decision to use the land for a hospital. This is a

potential reason for assessing the opportunity cost even if an appropriate direct cost measure could be developed.

Note though that it would make no sense to treat the acquisition costs of either a hospital or a home as the costs of using the assets for a period of time such as a year; rather, some way must be found to allocate the acquisition costs for these assets over their useful lives. The user cost approach is one possible means of achieving this cost allocation. However, housing markets are often not in equilibrium: a requirement for the derivation of the rental equivalent measure of OOH services using a user cost approach as in section 3.1. Thus, interest in determining the opportunity cost of OOH services, arises, in part at least, because of the lack of some other valid measure of the true economic cost involved.

5.2 The PPV Opportunity Cost Justification of the Rental Equivalence Approach

PPV state that the measurement question that must be addressed is:

“How much richer would the homeowner be if he or she did not consume the housing services provided by a dwelling?”

They go on to provide the following guidance on this opportunity cost calculation. They explain that a homeowner always has the option of moving out of his/her house in order to rent it to a tenant for rent r1, and of then moving into another rental unit whose rent is r2, with r2 < r1. Doing so would free up (r1 – r2) income for other uses. This demonstrates, they note, that a homeowner is, in effect, giving up income equal to r1 (not r2, and not r1-r2) if he/she occupies his/her own house. They go on to note that: “The fact that shelter services are considered essential for survival (‘one has to live somewhere’) is irrelevant to the argument.”

PPV argue that every owner occupier passed up the opportunity to rent out their home by choosing to occupy the home themselves. Owning and occupying, and owning and renting, a dwelling out are mutually exclusive alternatives. Treating this pair of options as the only relevant alternatives a homeowner faced provides an opportunity cost justification for the conventional rental equivalence approach: a justification with the advantage that it pertains to the use of the services of the durable asset -- the home -- for some period t. Also, this justification does not rest on an assumption that real estate markets are in equilibrium, unlike the usual theoretical justification of the user cost approach (outlined in section 3.1 above). The development of an opportunity cost justification for the rental equivalence approach by PPV is, in our view, an important step forward. The arguments they level against the user cost approach would otherwise apply as well to the rental equivalence approach if both are viewed as based on the same theoretical arguments.

A theoretical justification of the rental equivalent that does not entail imposing a housing market equilibrium condition means too that the rental equivalent is not invalidated by empirical evidence that rents and housing prices often seem to move quite differently. This is helpful since there are many reasons why house prices and rents may have differing developments over time.

Transactions costs can be substantial for real estate. Verbrugge (2008) suggests that “the large costs associated with real estate transactions would have prevented risk neutral investors from earning expected profits by using the transaction sequence buy, earn rent on property, sell, and would have prevented risk neutral homeowners from earning expected profits by using the transaction sequence sell, rent for one year, repurchase.”
Owners and renters are subject to differing sorts of uncertainty regarding changes over time in housing related expenses.\textsuperscript{18} And, because of agency problems, a landlord may not wish to customize a rental unit to the same extent as a homeowner. Also rental units may be subject to greater depreciation (Crone et al., 2009). Landlords seem to be bound too by rental market conventions to change rents infrequently. Rental rate stickiness has been shown empirically to be particularly important for continuing tenants.\textsuperscript{19} In addition, the tax treatment of owner occupiers and renters differs in many countries including the United States.\textsuperscript{20}

Also, the rental market for luxury homes is thin. Sometimes the owners of luxury homes want or need to rent out their homes. Luxury homes tend to be offered for rent mostly under conditions that limit the options of a renter. To find renters, the owners of luxury homes often must compete on price for tenants who would normally rent lower quality housing units and cannot afford to pay much more than what they normally would pay.\textsuperscript{21}

Yet another reason may be that a landlord’s horizon may be longer than for an owner occupier. When the landlord builds or buys a rental property, the landlord will want to set rents at least equal to the user cost. However, once the property is built or bought, the cost is sunk and supply and demand factors for rental properties could cause the market rents to diverge from the expected user costs. Evidence of this factor at work includes cycles in the construction of rental units; when the landlord’s user cost exceeds market rent, building of new rental properties slumps and vice versa when the landlord’s user cost is below current market rent.

\subsection{5.3 The Alternative Options Passed Up by Owner Occupiers}

In the Wikipedia example, the set of alternative options the city had to consider when choosing the hospital was known: the options were the proposals submitted before a fixed deadline (which included the option chosen). Now consider the problem of trying to measure the opportunity cost for an owner occupier. We can observe when a home owner has decided to continue to own and occupy their dwelling for the period (and perhaps for many periods to come). But what are the options that the owner occupier had, but passed up in choosing to continue to own and occupy their home?

PPV argue that each owner occupier in each period gave up the alternative of renting out their dwelling for that period. What we add to PPV’s contribution is the insight that most owner occupiers also gave up alternative financial investment opportunities. The opportunity cost of a choice taken is the value of the next best available alternative. Thus, to properly determine the financial investment component of the opportunity cost of a choice to own and occupy in period t,

\footnotesize{\textsuperscript{18} See Sinai and Souleles (2005).}
\footnotesize{\textsuperscript{19} See, for example, Gordon and Goethem (2004) and the findings of Genesove (2003). Also, Hoffmann and Kurz-Kim (2006, p. 5) report the following: “In our sample, prices last on average more than two years... but then change by nearly 10 \%. The longest price durations are found for housing rents, which, on average, are for more than four years.” Also, Hoffmann and Kurz-Kim (2006, p.5) report that German rents change only every 4 years on average.}
\footnotesize{\textsuperscript{20} See, for example, Poterba and Sinai (2008).}
\footnotesize{\textsuperscript{21} It also seems likely to us that, moving up the value scale, an increasing percentage of homes offered for rent are, in fact, offered with the terms of payment including house sitting duties along with the monetary rent obligations. Situations like this should, of course, be caught by the questions asked as part of the collection of the rent data, but it seems likely that not all the cases like this are properly identified.}
it is not necessary to know the entire alternative financial investment choice set that the owner occupier faced. Rather, it is only necessary to consider the highest valued of those alternatives. In addition, the alternatives considered as foregone opportunities must be mutually exclusive with each other and with the option chosen of owning and occupying in period t.22

5.4 The Financial User Cost of an Owner Occupier

The user cost approach is an appropriate way of allocating over time initial financial investments made in assets that yield earnings over multiple subsequent time periods. Unlike real estate markets, financial markets are generally believed to satisfy the assumptions on which the user cost derivation is based. Thus, we derive the financial user cost for an owner occupier with non-negative home equity at the start of period t in sub-section 5.4.1. We explore the nature of the financial user cost component of the OOH opportunity cost for a homeowner with non-negative home equity in various special circumstances in sub-section 5.4.2. We then take up the negative equity case in sub-section 5.4.3. Throughout section 5.4, we abstract from taxes, we assume there are no defaults, and we do not explicitly deal with home rental or purchase or financing or refinancing choices. Rather, we take the perspective of devising ways of accounting for OOH services in a CPI conditional on the product choices people have made, including their housing services consumption choices. This is the usual practice for price index construction.

5.4.1 Homeowners with Non-Negative Equity

In what follows, we proceed under the conjecture that the highest value of the forgone financial investment alternatives for an owner occupier with non-negative home equity is the expected earnings that would have resulted from investment of the home equity funds that the sale of the home at the start of period t would have freed up: a mutually exclusive alternative to the choice actually made of continuing to own and occupy and to the other alternative considered and rejected of owning and renting out the dwelling. A homeowner is viewed as having made the choice to continue owning their home with some level of debt \( D^t \geq 0 \) and equity \( V^t - D^t \) and with a known, required mortgage payment \( I^t \) due at the end of each time period,23 which can be expressed as \( I^t = (I^t / D^t)D^t = r_D^t D^t \) with \( r_D^t \) being defined by \( (I^t / D^t) = r_D^t \).

The financial user cost for owning the home in period t and living in it, discounted to the start of the period, is:

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22 There are multiple ways in which a homeowner might have withdrawn equity from their home. For example, refinancing lets a homeowner sell (or buy back) a fraction of an owned home. However, for the purposes of determining the opportunity cost of a choice taken, alternatives foregone that could have been jointly selected, such as renting the home out for period t and withdrawing some home equity during that time period too, must be considered as single, combined items in the set of mutually exclusive choices.

23 Mortgage contracts often contain fixed monthly payment requirements and also a fixed rate (or rates) of interest on the outstanding principal of the mortgage. The monthly payment typically represents a combination of interest and principal, but all that matters in our analysis is that the amount is fixed at the start of each period for the duration of m subsequent periods.
where $V_{t+1}$ is defined as the value of the home at the beginning of the period plus the expected per period average appreciation of the home over the $m$ subsequent time periods that the dwelling is assumed to be able to provide housing services. We recommend that $m$ should be set at a value at least as large as the median number of years that the relevant population of homeowners report having lived in their present homes.\(^{24}\) This implies that a very long term rate should be used for the expected home value appreciation. Now, if we multiply expression (5-1) through by the discount factor, $1 + r^t$, we obtain the following equivalent expressions for the end of period user cost:

\[
(5-2) \quad u^t = r_D^t D^t + r^t (V^t - D^t) + O^t - (V_{t+1}^t - V^t),
\]

\[
(5-3) \quad = O^t + (r_D^t - r^t) D^t - [V_{t+1}^t - (1 + r^t) V^t].
\]

### 5.4.2 Properties of the Financial User Cost for Homeowners with Non-Negative Equity

The user cost expression given in (5-3) can be better understood by considering some specific types of situations. Consider first of all a homeowner with no mortgage debt. For them, expression (5-3) reduces to

\[
(5-4) \quad u^t = O^t - [V_{t+1}^t - (1 + r^t) V^t].
\]

This expression is essentially the same as the customary user cost derived by Katz (2009) and shown in (3-4) in box 1 above, and specified by Verbrugge and shown in (3-5).

We next consider the extreme case of owner occupiers with positive equity whose mortgage payment rate equals their expected rate of return on financial holdings (i.e., $r_D^t = r^t$). In this case too, (5-3) reduces to (5-4) above. Thus, the conventional user cost expression implicitly assumes that homeowners with mortgages make payments such that $r_D^t$ equals their expected rate of return on alternative financial investments.

Among owner occupiers with positive home equity, well off households often can get mortgages with interest rates that are less than prime and moreover so that $r_D^t < r^t$. In this case, the user cost expression (5-3) can be written as:

\[
(5-6) \quad u^t = O^t - (r^t - r_D^t) D^t - [V_{t+1}^t - (1 + r^t) V^t],
\]

---

\(^{24}\) Most people own their homes for 7 years or more. In 2004, for example, there were 72 million owner-occupied homes and existing home sales were 6.8 million. Indeed, owner occupiers typically roll forward the equity accumulated in one owned home into another owned home when their housing needs change. Few return to being renters, even for brief periods. Many people move into their own owned homes as soon as they can afford to after reaching adulthood and die still owning their own homes.
where the term \((r^t - r_D^t)\) is now positive. Thus, all else equal, for these homeowners, \textit{higher mortgage debt reduces the financial user cost of OOH services}.

On the other hand, most subprime loans are high cost. For positive equity owner occupiers with \(r_D^t > r^t\), the user cost expression (5-3) reduces to:

\[
(5-7) \quad u^t = O^t + (r_D^t - r^t)D^t - [V^{t+1} - (1 + r^t)V^t],
\]

where \((r_D^t - r^t)\) is positive. All else equal, for these homeowners, \textit{higher mortgage debt means a higher financial user cost for OOH services}.

### 5.4.3 Homeowners with Negative Equity

We now turn our attention to the negative equity case: a case of special relevance in the wake of the recent burst of a housing market bubble. Right now, many households are living in homes that could not be sold for enough to cover their debts. Ashcraft and Schuermann (2008) cite data showing that second mortgages were quite rare in the 1990s, but that by 2006, many first mortgages were accompanied by second mortgages. In a comprehensive survey report of the Bank for International Settlements (a central bank forum), for the post-2000 period in the United States, Ellis (2008) calls attention to an increased use of second mortgages, and observes that many U.S. households were able to obtain 100 percent financing in this way. Ellis notes that, in addition to initial debt-to-value ratios being higher than before, the debt-to-value ratios for a growing proportion of home owners failed to decline over time as expected based on previous patterns, or even increased because of recent declines in home values, moving increasing numbers of home owners into negative home equity situations. Cagan (2007) estimates that around 5 percent of loans made in the early 2000s were already in negative equity at the end of 2006, though the figure for older loans was lower. Cagan estimates that around 18 percent of mortgages originated in 2006 were in negative equity by the end of that year.

An owner occupier who has borrowed using their home as collateral basically is renting part of their home from themselves and part from their creditor, with the rent that the creditor is charging (i.e., the interest and the required partial repayment of principal that must be made at the end of each time period for the owner to be able to either occupy or rent out the home) being specified in the mortgage contract. As before, the mortgage debt in nominal terms is denoted by \(D^t\), the required payment on the mortgage that is due at the end of period \(t\) is \(r_D^tD^t\), the beginning of the period \(t\) market value of the home is \(V^t\), and the expected end of period \(t\) market value is \(V^{t+1}\). Mortgage debt in the amount of \(D^t\) can consist of some combination of a conventional first mortgage and subprime or second mortgage funds. Negative equity at the beginning of period \(t\) means that debt \(D^t\) is larger than beginning of period \(t\) equity \(V^t\) so that

\[
(5-8) \quad V^t - D^t < 0, \text{ and hence } D^t - V^t > 0.
\]

If the homeowner had let go of their home at the beginning of period \(t\) by either selling it or “giving it back to the bank” for the current market value of \(V^t\), and if they either could not or chose not to consider simply walking away from their residual debt, then household net worth
would have had to decrease at the start of period \( t \) by the amount of the homeowner’s negative equity position at the start of \( t \), which is \( D^t - V^t > 0 \). Of course, there are differences in price levels at the beginning and end of the period and so the end of the period costs must be discounted relative to the beginning of the period costs. But what is the appropriate discount factor in this case?

If the homeowner has investments that are earning more than the mortgage payment rate \( r_D^t \), it would not make sense to liquidate these investments to pay off the negative equity. Nor would it make sense for the homeowner to borrow at a rate higher than \( r_D^t \) to pay off the negative equity.\(^{25}\) Rather, ignoring possible institutional or moral constraints, it would only make sense for the homeowner to borrow or to liquidate investments with an associated rate of less than \( r_D^t \) (say the rate of \( \rho^t \leq r_D^t \)). We let \( \rho^t \) denote a suitable discount rate for the homeowner. The cheapest funds for paying off negative equity at the start of \( t \), and hence the best financial alternative, are funds in a conventional savings or checking account that are earning no interest, or household savings achieved by cutting back on other beginning of period \( t \) household expenses, in which case the appropriate discount rate might be the anticipated Consumer Price Index Inflation rate over period \( t \), since discounting by one plus this rate will make asset values at the end of the period comparable to asset values at the beginning of the period in terms of consumption equivalents. To know the end of period value of the beginning of period negative equity, we need to know where those funds would have come from. In computing the net value of the most attractive way in which the homeowner could have let go of the home at the start of period \( t \), we must also allow for the fact that the end of period negative equity position, given continued ownership and occupation of the dwelling, is expected to be \( D^t - \frac{1}{1+\rho^t} V^{t+1} \), with the costs as well of \( r_D^t D^t \) and \( O^t \) due at the end of the period.

In what follows, we will assume that the appropriate period \( t \) discount rate, \( \rho^t \), is a rate at least as high as the anticipated CPI inflation rate and less than the mortgage payment rate of \( r_D^t \). Thus assuming that mortgage interest and operating costs are paid at the end of the period, we find that the beginning of the period user cost \( u^t/(1+\rho^t) \) is defined as follows:

\[
\frac{u^t}{1+\rho^t} = -[D^t - V^t] - \left[ -r_D^t D^t - O^t - (D^t - \frac{1}{1+\rho^t} V^{t+1}) \right] \frac{1}{1+\rho^t} \]

(5.9)

\[
= -[D^t - V^t] + \left[ r_D^t D^t + O^t + (D^t - \frac{1}{1+\rho^t} V^{t+1}) \right] \frac{1}{1+\rho^t} .
\]

Multiplying both sides of the above expression through by \((1+\rho^t)\) leads to the following expression for the end of period \( t \) user cost for owner occupied housing if beginning of the period equity is negative:

\(^{25}\) Some negative equity homeowners will have low interest mortgage options. However, it is typically the case that negative equity homeowners have relatively high interest mortgage debt.
The expression for the user cost in (5-11) says that user cost is equal to operating expenses $O^t$ plus real mortgage interest $(r_D^t - \rho^t)D^t$ less the anticipated real capital gain in the value of the house $[V^{t+1} - (1 + \rho^t)V^t]$. This expression makes sense intuitively.

We will not develop a user cost formula for a homeowner with negative equity who can walk away from paying off their debt with no consequences. The problem of transactions and adjustment costs becomes critical in this case. If there were no transactions and no other adjustment costs of buying and selling and building homes and there were no other consequences of defaulting, then in the negative equity case, it would make sense for the homeowner to default and immediately repurchase their home at the new lower price (or buy another home of equivalent value). However, with transactions and adjustment costs, it can make sense for the negative equity homeowner to stay in the home without defaulting. Thus the size of the transactions and adjustment costs becomes critical.

6. Inflation Measurement Given Dwelling Uniqueness

A decision by the BLS and other official statistics agencies to move to an opportunity cost approach for accounting for OOH in a CPI would be a step forward, in our view. However, regardless of whether an opportunity cost or a user cost or a rental equivalent approach is used, statistical agencies will still face empirical methodology problems arising from dwelling uniqueness. Each dwelling has a unique location and dwellings continually evolve via dwelling-specific depreciation and renovation. Dwelling uniqueness presents similar empirical challenges regardless of whether the price data are sale prices or rental prices. Thus, in this section, we use the term “price” to refer to either dwelling sale or rent observations, unless otherwise specified.

The depreciation rate for a dwelling can be defined and can hypothetically be measured by the ratio of the same period prices for identical dwellings that have been used for different lengths of time. The inflation rate for a dwelling can be defined and can hypothetically be measured by the ratio of the prices at different points in time for identical dwellings that were used the same length of time. However, no two dwellings are identical. Among other differences, they all have different physical locations. Cross sectional information on used asset sale prices or rents at any one point in time will not allow us to separate out the separate effects of depreciation and inflation for durable assets that must be viewed as unique for price measurement purposes.

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26 We have not modeled various tax consequences associated with home ownership. This is left to further research.

27 Special cases of this fundamental identification problem have been noted in the context of various econometric housing models: “For some purposes one might want to adjust the price index for depreciation. Unfortunately, a depreciation adjustment cannot be readily estimated along with the price index using our regression method…. In applying our method, therefore, additional information would be needed in order to adjust the price index for depreciation,” Bailey, Muth and Nourse (1963, p. 936). “The price index and depreciation are perfectly collinear, so if one cares about the price index, it is necessary to use external information on the geometric depreciation rate of houses,” Palmquist (2003, p. 43).
However, this separation can be based on empirical evidence if some way can be found for deciding when dwellings can be viewed as comparable for price measurement purposes.\footnote{For CPI rent measurement, BLS age adjusts inflation rates using the hedonic method of Randolph (1988a, 1988b).} Achieving this separation is especially important for any category of durables where maintenance and renovation expenditures and consequences are substantial, and also where inflation and deflation movements in prices can be substantial. The prospects for separating out depreciation and inflation effects are much improved if dwellings can be viewed as the same for price measurement purposes provided they have certain shared characteristics. This is true for the hedonic methods, as used for measuring inflation for both dwelling sale prices and dwelling rents, but not for the repeat sales method, which only compares the same unit over time. The repeat sales and hedonic methods are two seemingly very different types of empirical methods which we explain here can be placed within a common mathematical framework, thereby allowing users to take advantage of insights from research into the properties of both these methods.

With a pure hedonic method, data collected over multiple periods are classified by value determining characteristics that are not unique such as neighborhood or distance from the down town core, type of dwelling unit (e.g., single detached or a unit in a multiple unit building), some metric for size such as floor space, and the age of the property. To apply a hedonic method, data are needed on the selected list of value determining characteristics, and there must be agreement that this list is appropriate and adequate to control for differences in the value of the housing services provided by the different purchased or rented dwellings. Thus the hedonic method data requirements are usually very extensive. The inflation rate is then estimated using observations over time while controlling econometrically for changes in the value determining characteristics.

The repeat sales method compares the price observations for housing properties that were sold, or that were rented, multiple times over the time interval spanned by the available data.\footnote{The repeat sales procedure, now in widespread use, dates back to Bailey, Muth, and Nourse (1963). See also Dreiman and Pennington-Cross (2004) for the uses of this method and see Green and Malpezzi (2003, pp. 32-60) for a review of the repeat sales index literature.} This method is popular with U.S. real estate researchers and practitioners because it uses only the information readily available in all localities of the United States: sale or rental prices and the unique legal property descriptions. A key underlying assumption is that, with similar maintenance expenditures, owners of residential properties that they occupy or rent out usually manage to maintain their properties in unchanged condition over the dwelling service lives. This method does, however, control for the quality determining attributes of each dwelling that do not change over time, and it does so without the need for having data on (or even knowing about) all of those value determining attributes.

We now turn our attention to the specifications of the repeat sales and hedonic methods and how these methods are related. We begin with the repeat sales method which is due to Bailey, Muth and Nourse (1963). Since hedonic regression models, as usually used in the price measurement literature, have price levels (often in logarithmic form) as dependent variables, rather than price ratios as is the case for the repeat sales method, it is helpful to introduce the repeat sales method as it arose historically: as a generalization of the chained matched model methodology.\footnote{See Wyngarden (1927) and Wenzlick (1952).} This is the motivation for how the repeat sales method is presented in box 2.
Box 2. An Exposition of the Repeat Sales Method

Let S(0,1) denote the set of housing units that are in scope for the index and were sold in both periods 0 and 1. Denote the price for property n sold in period t by \( V_n^t \). Here attention is confined to just two time periods 0 and 1, so \( n \in S(0,1) \). Let \( P^{0,1} \) be the real estate price index going from period 0 to 1. For housing units in S(0,1), suppose the stochastic model relating the property sales price ratio, \( V_n^1 / V_n^0 \), to \( P^{0,1} \) is:

\[
V_n^1 / V_n^0 = P^{0,1} \exp \varepsilon_n^{0,1},
\]

where \( \varepsilon_n^{0,1} \) is assumed to be an independently distributed error term with mean 0 and constant variance. Taking logarithms of both sides of (4-1) leads to the following linear regression model:

\[
\ln[V_n^1 / V_n^0] = \pi^{0,1} + \varepsilon_n^{0,1},
\]

where \( \pi^{0,1} = \ln P^{0,1} \). The least squares estimator for \( \pi^{0,1} \) is the arithmetic average of the logarithms of the sales price ratios. Exponentiating this estimator yields a preliminary matched model property price index going from period 0 to 1:

\[
P^{0,1*} = \prod_{n \in S(0,1)} [V_n^1 / V_n^0]^{0,1} / N(0,1),
\]

where \( N(0,1) \) is the number of houses in the set S(0,1). This index is seen to be the equally weighted geometric mean of sales price ratios \( V_n^1 / V_n^0 \) for all the properties that changed hands in both periods 0 and 1: a typical matched model estimator for an elementary price index.

Next let \( N(1,2) \) denote the number of sales of houses in set S(1,2) and consider the set S(1,2) of houses that sold in both periods 1 and 2. Now the preliminary matched model price index going from period 1 to 2 can be shown to be:

\[
P^{1,2*} = \prod_{n \in S(1,2)} [V_n^2 / V_n^1]^{1,2} / N(1,2).
\]

Using the above results, the levels of the property price index, \( P^t \), for \( t = 0,1,2 \) can be defined as:

\[
P^0 = 1; P^1 = P^{0,1*}; P^2 = P^{1,2*}.
\]

Thus the price index \( P^t \) is set equal to 1 in period 0; in period 1, it equals the matched model price index going from period 0 to 1, and in period 2, it equals the product of the preliminary price indexes given in (4-3) and (4-4).

The Bailey, Muth and Nourse (1963) innovation was to reparameterize the model described above and to add an additional set of estimating equations for repeat sales pairs in periods 0 and 2; i.e., for housing properties in S(0,2). Their estimating equations with three periods of data on repeat sales are:

\[
\ln[V_n^1 / V_n^0] = \pi^1 - \pi^0 + \varepsilon_n^{0,1} \quad \text{for } n \in S(0,1),
\]
\[
\ln[V_n^2 / V_n^1] = \pi^2 - \pi^1 + \varepsilon_n^{1,2} \quad \text{for } n \in S(1,2),
\]
\[
\ln[V_n^2 / V_n^0] = \pi^2 - \pi^0 + \varepsilon_n^{0,2} \quad \text{for } n \in S(0,2),
\]

where now we have \( \pi^0 = \ln P^0 \), \( \pi^1 = \ln P^1 \), and \( \pi^2 = \ln P^2 \), with the following normalization imposed (where adding a constant to each \( \pi^t \) leaves the regression unchanged): \( \pi^0 = 0 \) or \( P^0 = 1 \). This leads to a model that can be estimated using least squares regression. Exponentiating the least squares estimates for the parameters \( \pi^1 \) and \( \pi^2 \), denoted here by \( \pi^{1*} \) and \( \pi^{2*} \), leads to estimates for the preliminary indexes \( P^{1*} \) and \( P^{2*} \). The BMN estimates for the housing price levels in periods 1-3 are:

\[
P^0 = 1, \quad P^1 = \exp \pi^{1*}, \quad P^2 = \exp \pi^{2*}.
\]

The 3-period model generalizes easily to the T-period case of Bailey, Muth and Nourse (1963).
As originally proposed, the repeat sales method can only be used to measure price level change over time, and thus is not well suited for empirical studies that seek to exploit cross-sectional variation by using the absolute dollar values of dwellings.\footnote{See, for example, Capozza, Hendershott, Mack, and Mayer (2002).} However, in the literature on spatial price level comparisons, Summers (1973) proposed a hedonic regression model where the only explanatory variables are dummy variables for the country and the product: the country-product-dummy or CPD method. In box 3, we show the formal steps for how the repeat sales and the CPD methods are related for the case where complete matched model data are available.

**Box 3. The CPD Model with Complete Matched Model Data**

Consider a sample of $N$ houses ($n=1,2,...,N$) each of which sold in each of the three periods ($t=0,1,2$): $S(0,1,2)$. A stochastic model for the house prices, $V^t_n$, in each period $t$ can be specified as follows:

\[
V^t_n = \alpha_n P^t \exp{\varepsilon^t_n}, \quad n=1,...,N,
\]

where $P^t$ is the housing price index level for period $t$, $\alpha_n$ is a parameter that reflects the quality of housing unit $n$ relative to “average” quality and $\varepsilon^t_n$ is an independently distributed, mean zero, constant variance error term. Taking logarithms of both sides of (4-12) leads to the following system of estimating equations for the $N$ houses:

\[
\ln V^t_n = \beta_n + \pi^t + \varepsilon^t_n, \quad n=1,...,N, \quad t=0,1,2.
\]

where $\beta_n = \ln \alpha_n$ and $\pi^t = \ln P^t$. The model defined by (4-13) and the normalization (4-14), the least squares (LS) estimators for the model parameters satisfy the following $N+2$ equations:

\[
\sum_{n=1}^{N} \ln V^1_n = \sum_{n=1}^{N} \beta^*_n + N\pi^0, \quad t=0,1,2.
\]

Using equations (4-16) to eliminate the $\beta^*_n$ from (4-14) and (4-15) yields solutions for the unknowns:

\[
\pi^* = (1/N)\sum_{n=1}^{N} \ln[V^1_n/V^0_n], \quad \pi^{2*} = (1/N)\sum_{n=1}^{N} \ln[V^2_n/V^0_n].
\]

After exponentiating these estimates, this complete information CPD model leads to the following geometric mean of the period 1 relative to the corresponding period 0 values as the estimate for the period 1 housing price level, $P^{1*}$, and the geometric mean of the period 2 values relative to the corresponding period 0 values as the estimate for $P^{2*}$:

\[
P^{1*} = \prod_{n=1}^{N} [V^1_n/V^0_n]^{1/N}, \quad P^{2*} = \prod_{n=1}^{N} [V^2_n/V^0_n]^{1/N}.
\]

Finally, the details are shown in box 4 for how the repeat sales approach can be modified to incorporate hedonic regression corrections for changes in observed dwelling characteristics between price observations. The resulting linear regression model (equations (4-19)-(4-22)) is the same as the two country CPD model (with incomplete information).\footnote{It is also identical to the two period Aizcorbe-Corrado-Doms (2001) dummy product hedonic regression model.} Exponentiating (4-27) reveals that this hedonic regression model leads to a period 0 to 1 price index that equals the
equally weighted geometric mean of the selling prices in period 1 divided by the geometric mean of the selling prices of the matched models in period 0.\textsuperscript{33}

In its basic form, the general time dummy hedonic method involves regressing the logarithm of the property sale price on the characteristics of the property and a time dummy variable for each period spanned by the estimation data set (except the omitted base period). Once the estimation has been completed, the time dummy coefficients can be exponentiated to create an index. Alternatively, using information on the characteristics of the properties sold, the data can be stratified and a separate regression can be run for each time period for specified classes of residential properties. Thus the hedonic regression method could be used to produce a family of indexes. Diewert, Heravi and Silver (2009) outline alternative formulations and establish the relationships among them.\textsuperscript{34}

7. The Structures and Land Decomposition Problem\textsuperscript{35}

Usually the logarithm of the purchase price is taken as the dependent variable in real estate price models. While this specification accords with the directly observable property price information, it is inconsistent with certain aspects of the structure and land components of the price of a property. Residential real estate usually involves both a structure and the land that the structure is built on (the site). To model this composite, consider a sample of dwelling units purchased at the beginning of period 0.

Suppose the purchase price of property \( n \) is \( p_n^1 \). The value for property \( n \) can be regarded as the sum of the (often unobserved) cost per square meter for the structure, times the floor space of the structure in square meters (denoted by \( A \) in box 5), plus the price per square meter of land (often not directly observed) times the area of the site in square meters (denoted by \( B \) in box 5). For period 0, the property value can be represented as in equation (6-1) in box 5, and for period \( t \), the value for this property can be represented as in equation (6-2).

The structure and land components are probably subject to different rates of inflation and depreciation. Indeed, land is often viewed as depreciating little if at all (though there can be depreciation of site infrastructure). The asset inflation and depreciation effects are embedded in the coefficients of \( A \) and \( B \). Estimating equations are given by (6-3) and (6-4). If data are also available for the characteristics of the structure and the land, then the pair of equations, (6-5) and (6-6), can be estimated instead. This model is flexible and provides a means of decomposing a property price index into structural and land components, though the model is nonlinear. Moreover, the rate of depreciation may vary across rental and owner-occupied property.

\textsuperscript{33} In a series of papers, Diewert (2002, 2003a, 2003b, 2003c, 2004, 2005a, 2005b, 2006a, 2006b), Diewert, Heravi and Silver (2007/2010), Silver (2003), and Silver and Heravi (2005) show how alternative specifications and weights can be used within the CPD framework to derive a number of known index number formulas. See also de Haan (2003), Silver (2003) and Silver and Hervari (2005). Diewert (2005b) shows that the unweighted indexes can be far from their weighted counterparts. Thus it is important to run appropriately weighted regressions.

\textsuperscript{34} Rao (2008) notes that the CPD method -- the weighted version in particular -- is being increasingly used in deriving spatial comparisons due to its ability to handle price quotations. See, for example, Aten and Menezes (2002), Heston and Atten (2002), Rao (2003, 2005), and Deaton, Friedman and Alatas (2004).

Next a model is considered where not every house must trade in each period for information about the house to be included in the analysis data set. In order to minimize notational complexities, Diewert (2003a) provides the following details for the case of two periods. Let $S(0,1)$ be the set of housing units that sold in both periods 0 and 1. Taking into account the normalization (4-10), the estimating equations corresponding to these houses are:

\[
\ln V_n^0 = \beta_n^* + \ln u_n^0, \quad \text{for } n \in S(0,1),
\]

\[
\ln V_n^1 = \beta_n + \pi^1 + \ln u_n^1, \quad \text{for } n \in S(0,1).
\]

Let $S(0,\sim 1)$ denote the set of housing units in the target population that sold in period 0 but not in period 1. The estimating equations for these observations are:

\[
\ln V_m^0 = \gamma_m + \ln v_m^0, \quad \text{for } m \in S(0,\sim 1),
\]

where $\gamma_m$ is the logarithm of the quality adjustment factor for the $m$th housing unit that sold in period 0 but not in period 1. Similarly, let $S(1,\sim 0)$ denote the set of housing units in the target population that sold in period 1 but not in period 0. The estimating equations for these observations are:

\[
\ln V_k^1 = \delta_k + \ln u_k^1, \quad \text{for } k \in S(1,\sim 0),
\]

where $\delta_k$ is the logarithm of the quality adjustment factor for the $k$th dwelling for which price information is available in period 1 but not 0.

Let $\pi_1^*, \beta_n^*, \gamma_m^*$ and $\delta_k^*$ denote the least squares (LS) estimates of the parameters $\pi_1, \beta_n, \gamma_m$ and $\delta_k$ that appear in (4-19)-(4-22). The stacked vector of dependent variables for equations (4-19)-(4-22) can be written as the sum of the vectors of exogenous variables times their corresponding least squares estimates plus the vector of least squares residuals. As noted above, the inner product of each exogenous vector with the vector of LS residuals is zero. Thus the LS estimators for the unknown parameters in the regression model must satisfy the following equations:

\[
\sum_{n \in S(0,1)} \ln V_n^1 + \sum_{k \in S(1,\sim 0)} \ln V_k^1 = \sum_{n \in S(0,1)} \beta_n^* + N(0,1)\pi_1^* + \sum_{k \in S(1,\sim 0)} \delta_k^* + N(1,\sim 0)\pi_1^*;
\]

\[
\ln V_n^0 + \ln V_n^1 = 2\beta_n^* + \pi_1^*, \quad \text{for } n \in S(0,1)
\]

\[
\ln V_m^0 + \ln V_m^1 = \gamma_m^*, \quad \text{for } m \in S(0,\sim 1),
\]

\[
\ln V_k^1 = \delta_k^*, \quad \text{for } k \in S(1,\sim 0),
\]

where $N(0,1)$ is the number of dwellings that traded in both periods and $N(1,\sim 0)$ is the number that sold in 1 but not 0. Equations (4-26) can be used to eliminate the $\delta_k^*$ in equation (4-23), and equations (4-24) can be used to eliminate the $\beta_n^*$ from equation (4-23). The resulting equation for $\pi_1^*$ is:

\[
\pi_1^* = [1/N(0,1)]\sum_{n \in S(0,1)} \ln[V_n^1 / V_n^0],
\]

which is the arithmetic average of the logarithms of the sales price ratios for the two periods.

For the housing units that sold (or were rented) in $t$, a more general hedonic regression model is:

\[
\ln V_n^t = \pi_1 + \sum_{k=1}^{K} z_{nk}^t \beta_k + \varepsilon_n^t, \quad n \in S(t).
\]

$\varepsilon_n^t$ is an independently distributed error term with mean 0 and constant variance, $V_n^t$ is the observed selling price or rent of dwelling $n$ in period $t$, $z_{nk}^t$ is the amount of characteristic $k$ that dwelling $n$ has, and $\pi_1^*$ equals the logarithm of the constant quality price index, $P_1^*$; i.e., $\pi_1^* = \ln P_1^*$ for $t = 0,1,\ldots,T$. The parameter $\beta_k$ transforms amounts of characteristic $k$, $z_k$, into constant quality utility units for $k = 1,\ldots,K$. 


Suppose the total cost, $p$, of a property after the structure is completed will equal the floor space area of the structure, say $A$ square meters, times the building cost per square meter, $\alpha$ say, plus the cost of the land, which will equal the cost per square meter, $\beta$ say, times the area of the land site, $B$. Now think of a sample of properties of the same general type, with prices, $p_n$, in period 0 and structure areas $A_n$ and land areas $B_n$ for $n=1,...,N(0)$, and where these prices are equal to costs of the above type times error terms $\eta_n$ which have mean 1. This leads to a hedonic regression model for period 0 where $\alpha$ and $\beta$ are the parameters to be estimated in the regression:

$$ p_n^0 = [\alpha A_n^0 + \beta B_n^0] \eta_n^0. $$

Taking logarithms of both sides of (6-1) leads to the following traditional additive errors regression model:

$$ \ln p_n^0 = \ln[\alpha A_n^0 + \beta B_n^0] + \epsilon_n^0, $$

where the new error terms, $\epsilon_n^0 = \ln \eta_n^0$ for $n=1,...,N(0)$, are assumed to have 0 means and constant variances.

For a subsequent period $t$, the price per square meter for the given type of structure will have changed from $\alpha$ to $\alpha^t$ and the land cost per square meter will have changed from $\beta$ to $\beta^t$ where $\gamma^t$ is the period 0 to $t$ price index for the type of structure and $\delta^t$ as the period 0 to $t$ price index for the land that is associated with this type of structure. For $n=1,...,N(t)$, the period $t$ counterparts to (6-1) and (6-2) are:

$$ p_n^t = [\alpha^t A_n^t + \beta^t B_n^t] \eta_n^t, $$

and

$$ \ln p_n^t = \ln[\alpha^t A_n^t + \beta^t B_n^t] + \epsilon_n^t, $$

where $\epsilon_n^t = \ln \eta_n^t$, the period $t$ property prices are $p_n^t$, and the structure and land areas are $A_n^t$ and $B_n^t$.

Diewert (2006a) suggests that equations (6-2) and (6-4) can be run as a system of nonlinear hedonic regressions. The main parameters of interest are $\gamma^t$ and $\delta^t$, which can be interpreted as period $t$ price indexes (relative to the corresponding period 0 price levels of 1) for the price of a square meter of this type of structure and the price per meter squared of the underlying land.

This framework can be generalized to encompass the traditional array of characteristics used in real estate hedonic regressions. Suppose that we can associate with each property $n$ that is transacted in $t$ a list of $K$ price determining characteristics $X_{n1}^t, X_{n2}^t,...,X_{nK}^t$ for the structure and a similar list of $M$ price determining characteristics $Y_{n1}^t, Y_{n2}^t,...,Y_{nM}^t$ for the type of land. The equations that generalize (6-2) and (6-4) are:

$$ \ln p_n^0 = \ln[(\alpha_0 + \sum_{k=1}^{K} X_{nk}^0 \alpha_k) A_n^0 + [\beta_0 + \sum_{m=1}^{M} Y_{nm}^0 \beta_m] B_n^0] + \epsilon_n^0, \quad n=1,...,N(0) $$

and

$$ \ln p_n^t = \ln[(\alpha_0 + \sum_{k=1}^{K} X_{nk}^t \alpha_k) A_n^t + \delta^t \sum_{m=1}^{M} Y_{nm}^t \beta_m] B_n^t] + \epsilon_n^t, \quad n=1,...,N(t), $$

where the parameters to be estimated are now the K+1 quality of structure parameters, $\alpha_0, \alpha_1,...,\alpha_K$, the $M+1$ quality of land parameters, $\beta_0, \beta_1,...,\beta_M$, the period $t$ price index for structures parameter $\gamma^t$ and the period $t$ price index for the land underlying the structures parameter $\delta^t$. Note that $[\alpha_0 + \sum_{k=1}^{K} X_{nk}^0 \alpha_k]$ in (6-5) and (6-6) replaces the single structures quality parameter $\alpha$ in (6-2) and (6-4) and $[\beta_0 + \sum_{m=1}^{M} Y_{nm}^0 \beta_m]$ in (6-5) and (6-6) replaces the single land quality parameter $\beta$ in (6-2) and (6-4).
8. Concluding Remarks

The recent housing bubble has raised several questions for the measurement of housing service inflation. The most fundamental question is, are rents always the best measure of the opportunity cost of owner-occupied housing? Since, as Verbrugge has shown, rents often diverge from conventional user cost measures, the possibility has arisen that the answer is no.

Also, the markets for rental and owner occupied housing units can be quite isolated from one another. This is evident in Crone et al.’s hedonic measures of housing services for owner-occupied housing. Heston and Nakamura (2009) show too that for more expensive homes in the United States, the estimated rent/price ratio is substantially lower, suggesting that the housing services of more expensive homes are being underestimated in the U.S. national expenditures as these are being evaluated now. The equilibrium conditions under which rents equal user costs may not be fulfilled for sustained periods of time, either because the rental market is thin in some areas and price ranges where there are substantial proportions of owner occupied housing, or because of disequilibria due to other factors including the time required for new housing construction and government regulations governing land use and building that are believed to make the housing market prone to bubbles.

We have explored one comprehensive measure of housing services that may remain valid or may better approximate the trend rate of housing services cost inflation during conditions in which rents and user costs diverge. Our proposed measure of the opportunity cost of housing services is the greater of rents and user costs. This measure not only arguably permits more accurate measurement of housing services in disequilibrium, but also avoids the difficult problem of the possibility that user cost in the short run may be negative.

It is worth remarking that the difficulty of measuring housing services arises from three sources: (1) homes are unique, because of location, (2) homes are durable assets, and (3) homeowner transaction and housing supply adjustment costs are large. With large transaction and adjustment costs, prices may remain out of equilibrium because arbitrage is too costly. As durable assets, the rate of depreciation and the rate of interest, enter nontrivially. And because of uniqueness, prices of different units are not easily comparable. We have explored the inter-relationships of the repeat sales and hedonic methods for dealing with the comparability problem.

Owner-occupied housing services are in many countries the largest item in the consumer basket. In the United States consumer price index, it accounts for 24 percent of the total weight. How we measure inflation in this item is highly consequential for our understanding of macroeconomic dynamics, monetary policy, and growth.

References


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